

**Solutions to Problem 1.**

a.  $\Pr\{R_6 \leq 2\} = 1 - e^{-40(1/30)} \approx 0.7364$  ( $R_6 \sim \text{Exponential}(\lambda = 40)$ )

b.  $\Pr\{T_{300} \leq 6 \mid Y_2 = 150\} = \Pr\{Y_6 \geq 300 \mid Y_2 = 150\}$   
 $= \Pr\{Y_6 - Y_2 \geq 150 \mid Y_2 = 150\}$   
 $= \Pr\{Y_6 - Y_2 \geq 150\}$   
 $= \Pr\{Y_4 \geq 150\}$   
 $= 1 - \sum_{j=0}^{149} \frac{e^{-40 \cdot 4} (40 \cdot 4)^j}{j!} \approx 0.7956$

c.  $E[Y_{16} \mid Y_6 = 250] = 250 + E[Y_{16} - Y_6 \mid Y_6 = 250]$   
 $= 250 + E[Y_{16} - Y_6]$   
 $= 250 + E[Y_{10}]$   
 $= 250 + 40(10) = 650$

d. The first class passengers arrive according to a Poisson process with arrival rate  $\lambda_1 = 0.15(40) = 6$ .

$$E[T_{1,60}] = \frac{60}{\lambda_1} = 10$$

e.  $\Pr\{Y_{1,1} \geq 5 \text{ and } Y_{1,16} - Y_{1,15} \geq 5\} = \Pr\{Y_{1,1} \geq 5\} \Pr\{Y_{1,16} - Y_{1,15} \geq 5\}$  (independent increments)  
 $= \Pr\{Y_{1,1} \geq 5\} \Pr\{Y_{1,1} \geq 5\}$  (stationary increments)  
 $= (1 - \Pr\{Y_{1,1} \leq 4\})^2$   
 $= \left(1 - \sum_{j=0}^4 \frac{e^{-6(1)} (6(1))^j}{j!}\right)^2$   
 $\approx 0.5111$

**Solutions to Problem 2.**

a. If  $0 \leq \tau < 6$ :  $\Lambda(\tau) = \int_0^\tau 10 dt = 10\tau$

If  $6 \leq \tau \leq 12$ :  $\Lambda(\tau) = \int_0^6 10 dt + \int_6^\tau 4 dt = 4\tau + 36$

$$\Rightarrow \Lambda(\tau) = \begin{cases} 10\tau & \text{if } 0 \leq \tau < 6 \\ 4\tau + 36 & \text{if } 6 \leq \tau \leq 12 \end{cases}$$

b.  $E[Z_5 - Z_2] = \Lambda(5) - \Lambda(2) = 12$  ( $Z_5 - Z_2 \sim \text{Poisson}(\Lambda(5) - \Lambda(2)) = \text{Poisson}(12)$ )

c.  $\Pr\{Z_5 - Z_2 \leq 15 \mid Z_2 = 30\} = \Pr\{Z_5 - Z_2 \leq 15\}$   
 $= \sum_{j=0}^{15} \frac{e^{-12} (12)^j}{j!} \approx 0.8444$

**Solutions to Problem 3.**

In order for an arrival counting process (with arrivals one-at-a-time) to be Poisson, it must satisfy:

- Independent increments: in this context, the number of phone calls arriving at the cell tower in non-overlapping time intervals must be independent
- Stationary increments: in this context, the number of phone calls arriving at the cell tower only depends on the length of the interval, not when the time interval occurs